10CS34

USN

Third Semester B.E. Degree Examination, June/July 2018
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: *Answer FIVE full questions, selecting at least TWO questions from each part.*

PART – A

1 a. Let U be the set of real numbers, A = {x/x is a solution of $x^2-4=0$} and B = {–1,4} then compute
(i) $\overline{A}$ (ii) $\overline{B}$ (iii) $\overline{A \cup B}$ (iv) $\overline{A \cap B}$ **(08 Marks)**

b. Among 100 students in a class 32 study Maths, 20 study Physics , 45 study Biology, 7 study Maths and Physics, 10 study physics and Biology, 15 Study Maths and Biology, 30 do not study any one of. Then find
(i) The number of students studying all subjects.
(ii) The number of students studying exactly one subject. **(08 Marks)**

c. A fair coin is tossed 5 times. What is the probability that the number of heads always exceeds the number of tails as each outcome is observed. **(04 Marks)**

2 a. There are two restaurants next to each other. One has the sign that says "Good food is not cheap" and the other has a sign that says "cheap food is not good". Are the signs says same thing? If yes verify the answer. **(06 Marks)**

b. What is the difference between Tautology and Contingency? **(02 Marks)**

c. Verify the following without using truth table: $[(p \to q) \wedge (\neg r \vee s) \wedge (p \vee r)] \therefore \neg q \to s$. **(04 Marks)**

d. Write the negation of the following statements :
(i) If Rajiv is not sick, then if he goes to the picnic, then he will have a good time.
(ii) Ajay will not win the game or he will not enter the contest. **(08 Marks)**

3 a. Let n be an integer. Prove that n is odd if and only if 7n+8 is odd. **(08 Marks)**

b. Prove that for all real numbers x and y, if $x+y \geq 100$, then $x \geq 50$ or $y \geq 50$. **(08 Marks)**

c. For all positive real numbers x and y if the product xy exceeds 25, then x > 5 or y > 5. **(04 Marks)**

4 a. Prove that for any positive integer n,

$$\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$$

By mathematical induction, where F_n denote n^{th} Fibonacci number. **(08 Marks)**

b. Consider an 8×8 Chessboard. I contains 64 1×1 squares and one 8×8 square. How many 2×2 square does it contains? How many 3×3 squares? How many squares in total? **(12 Marks)**

PART – B

5 a. For each of the following functions, determine whether it is one-to-one and also determine its range,
(i) $f: z \to z,\ f(x) = x^3 - x$
(ii) $f: R \to R,\ f(x) = e^x$
(iii) $f: [0, \pi] \to R,\ f(x) = \sin x$ (06 Marks)

b. Prove that $\sum_{K=0}^{n} (-1)^K \binom{n}{n-K} (n-K)^m = 0$ for n = 5 and m = 2, 3, 4 (06 Marks)

c. Let f, g, $h: z \to z$ be defined by $f(x) = x - 1,\ g(x) = 3x,\ h(x) = \begin{cases} 0, & x \text{ Even} \\ 1, & x \text{ Odd} \end{cases}$ then determine
(i) $(f \circ g) \circ h$ (ii) g^3 (iii) f^3 (iv) h^{500} (08 Marks)

6 a. Determine the number of relations on A = {a, b, c, d, e} that are
(i) Antisymmetric (ii) Irreflexive (iii) Reflexive
(iv) Niether reflexive nor irreflexive. (08 Marks)

b. Draw the Hasse diagram for all the positive integer division of 72. (06 Marks)

c. How many of the equivalence relations on A = {a, b, c, d, e, f} have
(i) One equivalence class of size 4.
(ii) At least one equivalence class with three or more elements? (06 Marks)

7 a. State and prove Lagranges theorem for finite group G. (06 Marks)

b. Let G be a group with subgroups H and K. If $|G| = 660$, $|K| = 66$ and $K \subset H \subset G$, then what are the possible values for $|H|$? (06 Marks)

c. Define a cyclic group? Verify that $(Z_5^*, *)$ is cyclic. Find a generator of this group. (08 Marks)

8 a. Define addition and multiplication, denoted by ⊕ and ⊙ respectively, on the set Q as follows. For a, b ∈ Q, a ⊕ b = a + b + 7, a ⊙ b = a + b + (ab/7) then prove that(Q, ⊕ ,⊙) is a ring. (08 Marks)

b. Prove that, a finite integral domain (D, +, *) is a fild. (06 Marks)

c. If 3 distinct integers are randomly selected from the set {1, 2, 3, …., , 1000}. What is the probability that their sum is divisible by 3? (06 Marks)

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